

EE 740

Power Transformers

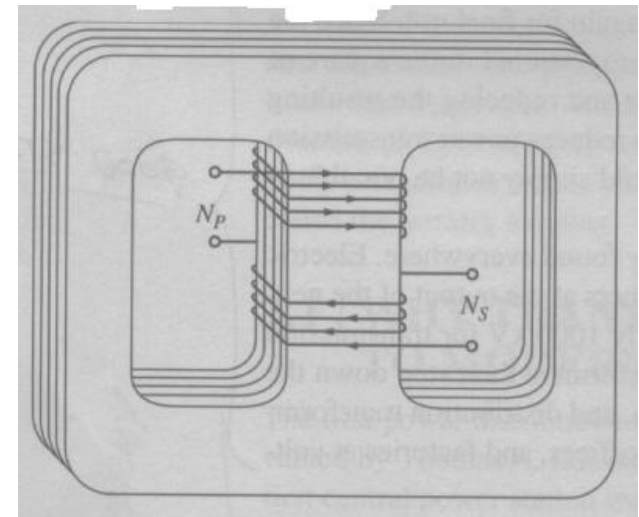
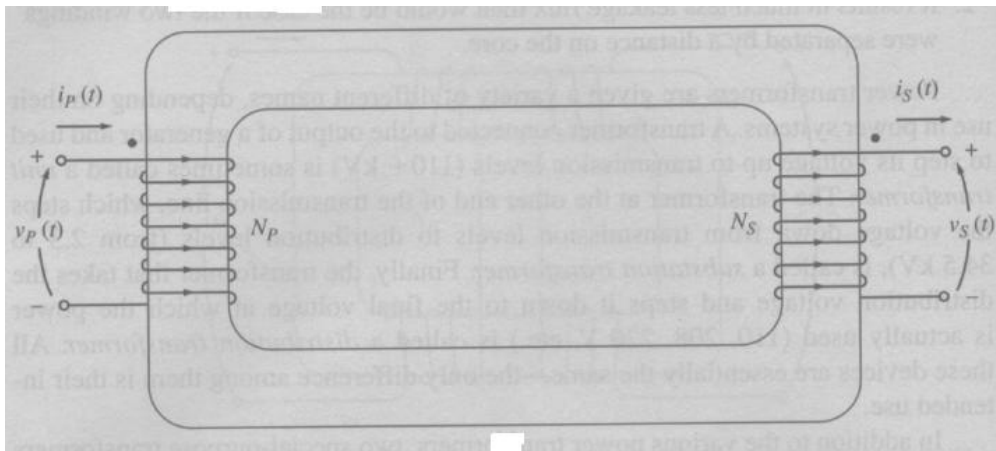


Spring 2013

Preliminary considerations

A transformer is a device that converts one AC voltage to another AC voltage at the same frequency. It consists of one or more coil(s) of wire wrapped around a common ferromagnetic core.

- In practice, the windings are wrapped on top of each other to minimize flux leakage.



Ideal transformer

An ideal transformer (unlike the real one) can be characterized as follows:

1. The core has no hysteresis nor eddy currents.
2. The magnetization curve is vertical with no saturation
3. The leakage flux in the core is zero.
4. The resistance of the windings is zero.

Consider a lossless transformer with an input (primary) winding having N_p turns and an output (secondary) winding of N_s turns.

The relationship between the voltage applied to the primary winding $v_p(t)$ and the voltage produced on the secondary winding $v_s(t)$ is

$$\frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a$$

where a is the turn ratio of the transformer.

Ideal transformer

The relationship between the primary $i_p(t)$ and secondary $i_s(t)$ currents is

$$\frac{i_p(t)}{i_s(t)} = \frac{1}{a}$$

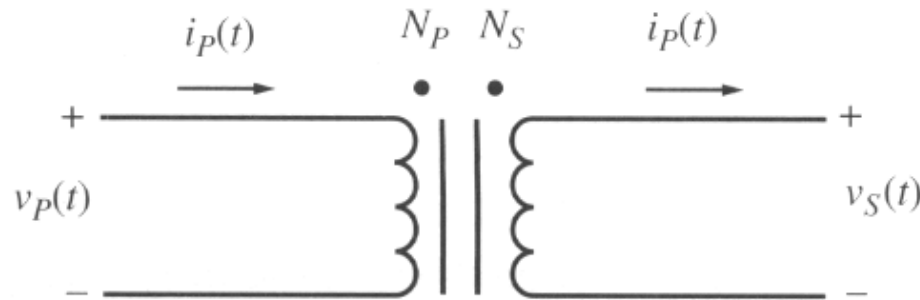
Phasor notation:

$$\frac{\mathbf{V}_p}{\mathbf{V}_s} = a$$

$$\frac{\mathbf{I}_p}{\mathbf{I}_s} = \frac{1}{a}$$

- The phase angles of primary and secondary voltages are the same.
- The phase angles of primary and secondary currents are the same also.
- The ideal transformer changes magnitudes of voltages and currents but not their angles.

Ideal Transformer



- One winding's terminal is usually marked by a dot used to determine the polarity of voltages and currents.
- If the voltage is positive at the dotted end of the primary winding at some moment of time, the voltage at the dotted end of the secondary winding will also be positive at the same time instance.
- If the primary current flows into the dotted end of the primary winding, the secondary current will flow out of the dotted end of the secondary winding.

Power in an ideal transformer

Assuming that θ_p and θ_s are the angles between voltages and currents on the primary and secondary windings respectively, the power supplied to the transformer by the primary circuit is:

$$P_{in} = V_p I_p \cos \theta_p$$

The power supplied to the output circuit is

$$P_{out} = V_s I_s \cos \theta_s$$

Since ideal transformers do not affect angles between voltages and currents:

$$\theta_p = \theta_s = \theta$$

Power in an ideal transformer

Since for an ideal transformer the following holds:

$$V_s = \frac{V_p}{a}; \quad I_s = aI_p$$

Therefore:

$$P_{out} = V_s I_s \cos \theta = \frac{V_p}{a} a I_p \cos \theta = V_p I_p \cos \theta = P_{in}$$

The output power of an ideal transformer equals to its input power – to be expected since assumed no loss. Similarly, for reactive and apparent powers:

$$Q_{out} = V_s I_s \sin \theta = V_p I_p \sin \theta = Q_{in}$$

$$S_{out} = V_s I_s = V_p I_p = S_{in}$$

Impedance transformation

The impedance is defined as a following ratio of phasors:

$$Z_L = \mathbf{V}_L / \mathbf{I}_L$$

A transformer changes voltages and currents and, therefore, an apparent impedance of the load that is given by

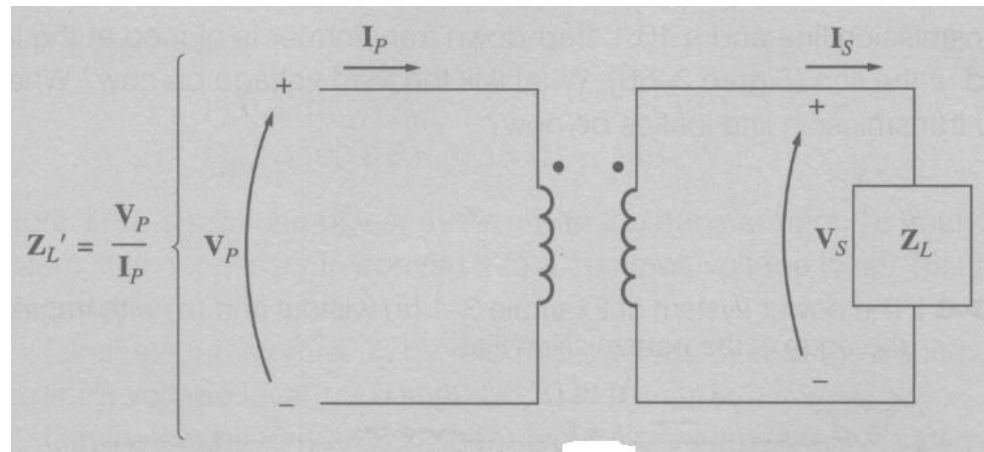
$$Z_L = \mathbf{V}_s / \mathbf{I}_s$$

The apparent impedance of the primary circuit is:

$$Z_L' = \mathbf{V}_p / \mathbf{I}_p$$

which is

$$Z_L' = \frac{\mathbf{V}_p}{\mathbf{I}_p} = \frac{a\mathbf{V}_s}{\mathbf{I}_s/a} = a^2 \frac{\mathbf{V}_s}{\mathbf{I}_s} = a^2 Z_L$$

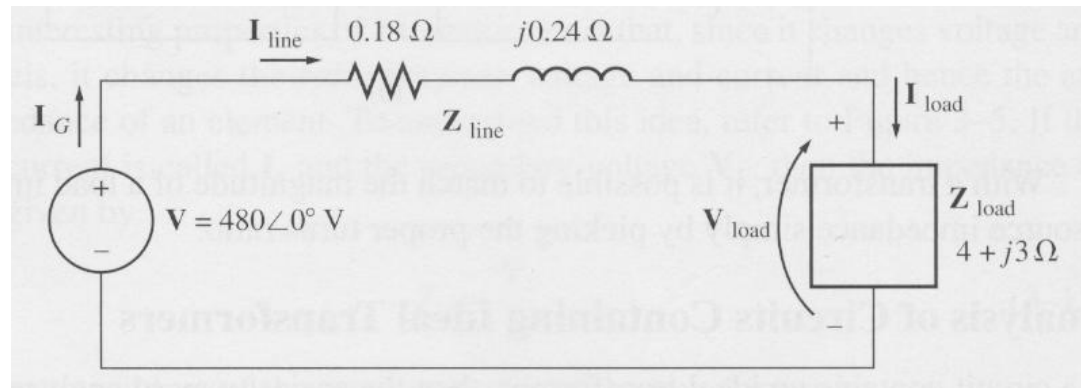


Analysis of circuits containing ideal transformers: Example

Example 4.1: a) What is the voltage at the load? Calculate the transmission line losses? b) If a 1:10 step up transformer and a 10:1 step down transformer are placed at the generator and the load ends of the transmission line respectively, what are the new load voltage and the new transmission line losses?

a) Without transformers:

$$\begin{aligned} I_G = I_{line} = I_{load} &= \frac{V}{Z_{line} + Z_{load}} \\ &= \frac{480 \angle 0^\circ}{0.18 + j0.24 + 4 + j3} \\ &= \frac{480 \angle 0^\circ}{5.29 \angle 37.8^\circ} = 90.8 \angle -37.8^\circ \text{ A} \end{aligned}$$



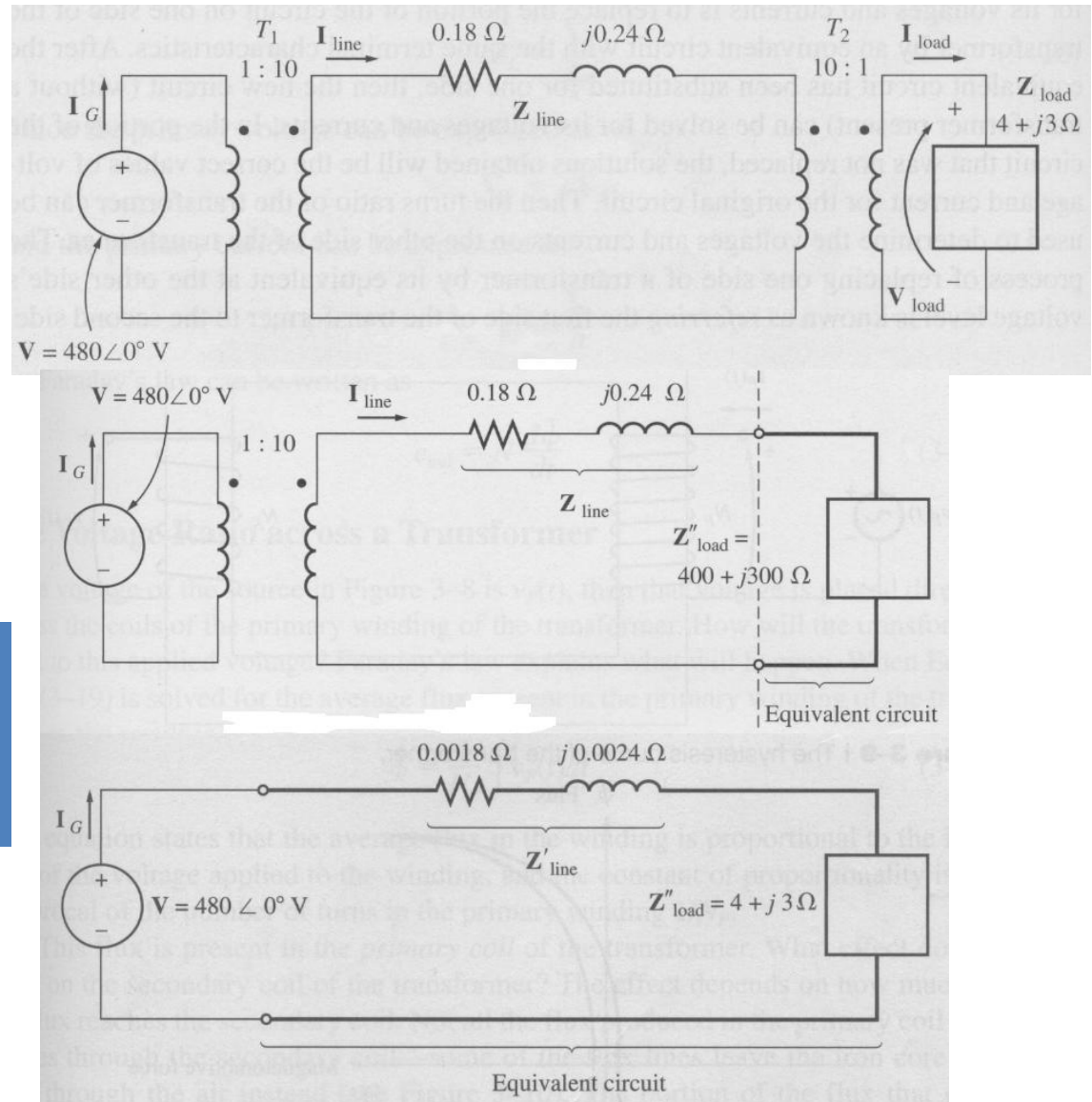
$$V_{load} = I_{load} Z_{load} = (90.8 \angle -37.8^\circ)(4 + j3) = (90.8 \angle -37.8^\circ)(5 \angle 36.9^\circ) = 454 \angle -0.9^\circ \text{ V}$$

$$P_{loss} = I_{line}^2 R_{line} = 90.8^2 \cdot 0.18 = 1484 \text{ W}$$

Analysis of circuits containing ideal transformers: Example

- b) With transformers, we will
- eliminate transformer T_2 by referring the load over to the transmission line's voltage level.
 - Eliminate transformer T_1 by referring the transmission line's voltage level to the source side,

$$\mathbf{I}_G = \frac{\mathbf{V}}{\mathbf{Z}'_{eq}} = \frac{480 \angle 0^\circ}{5.003 \angle 36.88^\circ} = 95.94 \angle -36.88^\circ \text{ A}$$



Analysis of circuits containing ideal transformers: Example

Knowing transformers' turn ratios, we can determine line and load currents:

$$\mathbf{I}_{line} = a_1 \mathbf{I}_G = 0.1 \cdot (95.94 \angle -36.88^\circ) = 9.594 \angle -36.88^\circ \text{ A}$$

$$\mathbf{I}_{load} = a_2 \mathbf{I}_{line} = 10 \cdot (9.594 \angle -36.88^\circ) = 95.94 \angle -36.88^\circ \text{ A}$$

Therefore, the load voltage is:

$$\mathbf{V}_{load} = \mathbf{I}_{load} \mathbf{Z}_{load} = (95.94 \angle -36.88^\circ)(5 \angle -36.87^\circ) = 479.7 \angle -0.01^\circ \text{ V}$$

The losses in the line are:

$$P_{loss} = I_{line}^2 R_{line} = 9.594^2 \cdot 0.18 = 16.7 \text{ W}$$

Real transformer

Flux leakage

A portion of the flux produced in the primary coil passes through the secondary coil (mutual flux); the rest passes through the external medium (leakage flux):

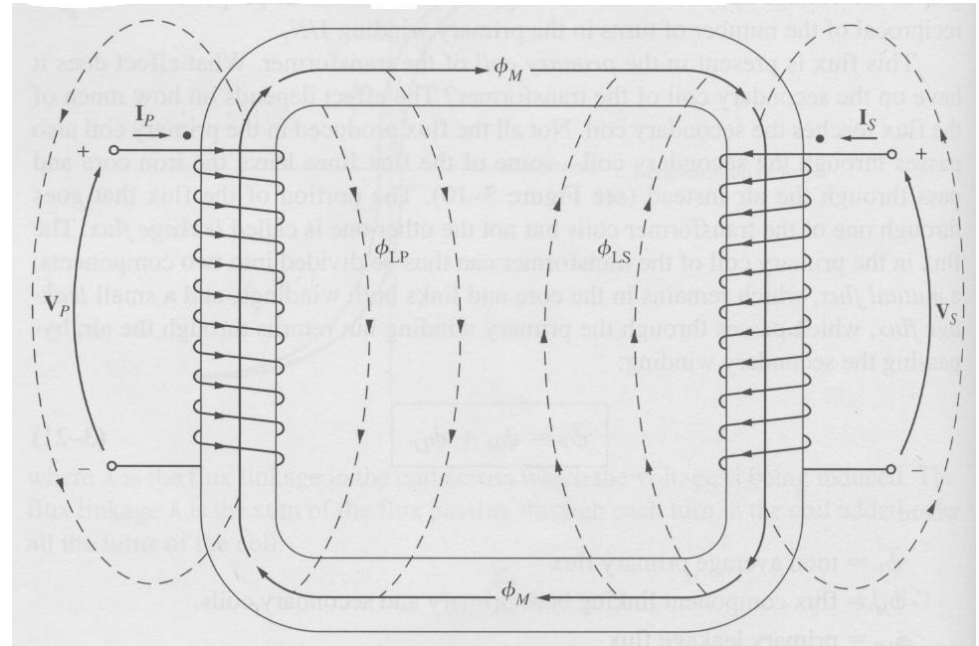
$$\bar{\phi}_p = \phi_m + \phi_{LP}$$

mutual flux leakage primary flux

Similarly, for the secondary coil:

$$\bar{\phi}_s = \phi_m + \phi_{LS}$$

Leakage secondary flux



Real transformer

From the Faraday's law, the primary coil's voltage is:

$$v_p(t) = N_p \frac{d\bar{\phi}_p}{dt} = N_p \frac{d\phi_m}{dt} + N_p \frac{d\phi_{Lp}}{dt} = e_p(t) + e_{Lp}(t)$$

The secondary coil's voltage is:

$$v_s(t) = N_s \frac{d\bar{\phi}_s}{dt} = N_s \frac{d\phi_m}{dt} + N_s \frac{d\phi_{Ls}}{dt} = e_s(t) + e_{Ls}(t)$$

The primary and secondary voltages due to the mutual flux are:

$$e_p(t) = N_p \frac{d\phi_m}{dt}$$

$$e_s(t) = N_s \frac{d\phi_{Ls}}{dt}$$

Combining the last two equations:

$$\frac{e_p(t)}{N_p} = \frac{d\phi_m}{dt} = \frac{e_s(t)}{N_s}$$

Real transformer

Therefore:

$$\frac{e_p(t)}{e_s(t)} = \frac{N_p}{N_s} = a$$

That is, the ratio of the primary voltage to the secondary voltage both caused by the mutual flux is equal to the turns ratio of the transformer.

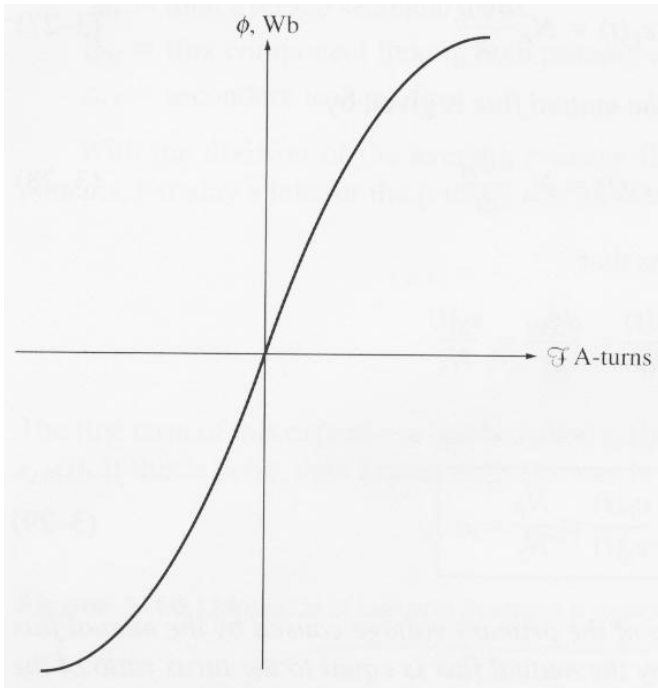
The following approximation normally holds since the leakage flux is much smaller than the mutual flux;:

$$\frac{v_p(t)}{v_s(t)} \approx \frac{N_p}{N_s} \approx a$$

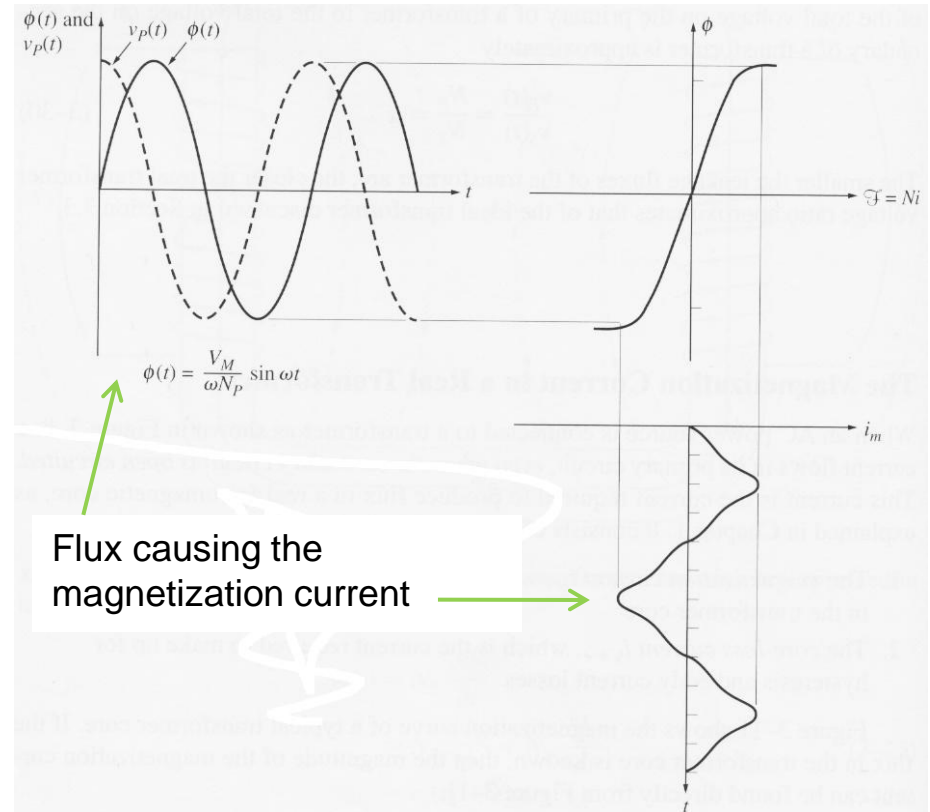
The magnetization current in a real transformer

Even when no load is connected to the secondary coil of the transformer, a current will flow in the primary coil. This current consists of:

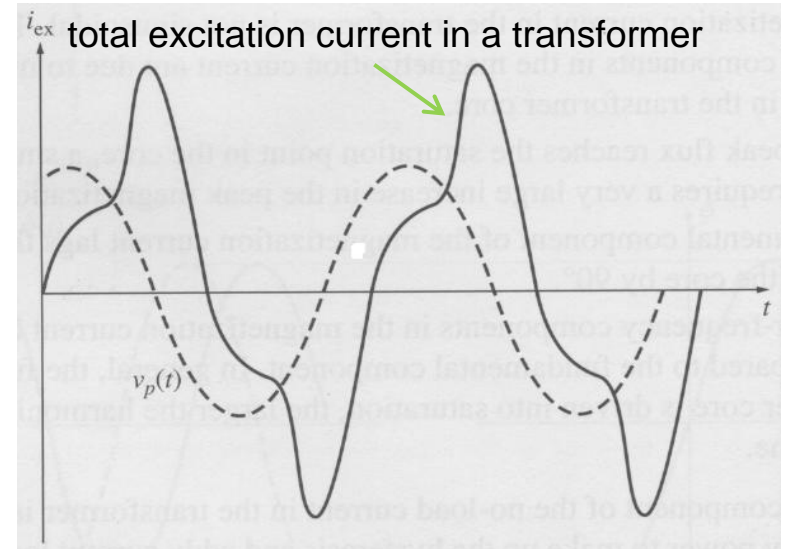
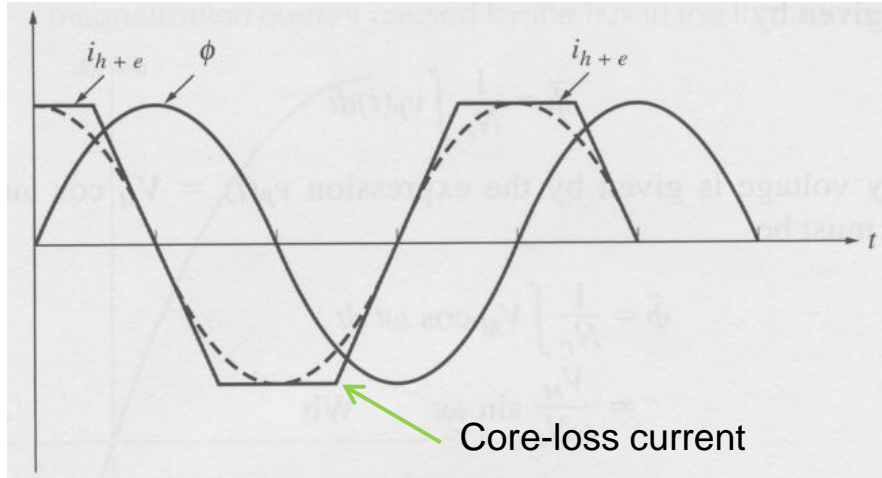
1. The magnetization current i_m is needed to produce the flux in the core;
2. The core-loss current i_{h+e} corresponds to hysteresis and eddy current losses.



Typical magnetization curve



Excitation Current in a real transformer



Core-loss current is:

1. Nonlinear due to nonlinear effects of hysteresis;
2. In phase with the voltage.

The total no-load current in the core is called the excitation current of the transformer:

$$i_{ex} = i_m + i_{h+e}$$

The current ratio on a transformer

If a load is connected to the secondary coil, there will be a current flowing through it.

A current flowing into the dotted end of a winding produces a positive magnetomotive force F :

$$F_p = N_p i_p$$

$$F_s = N_s i_s$$

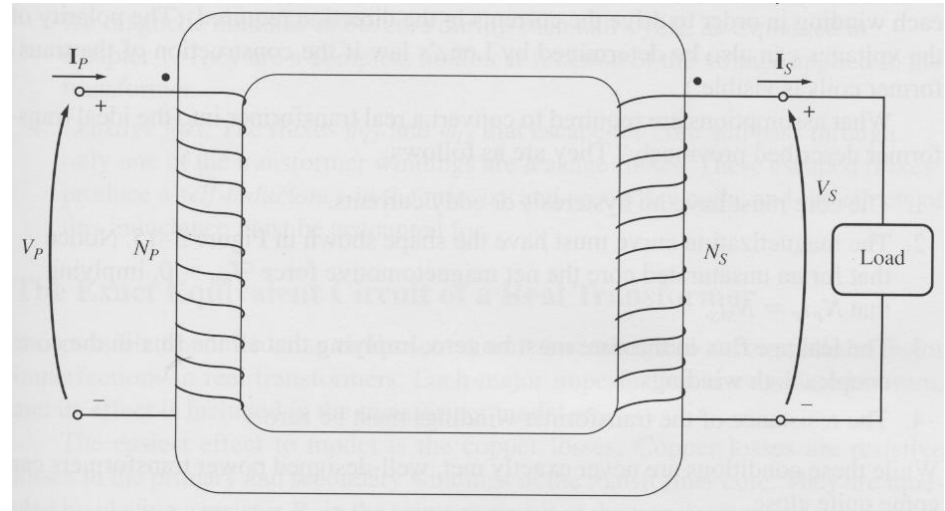
The net magnetomotive force in the core is

$$F_{net} = N_p i_p - N_s i_s = \phi \mathcal{R}$$

For well-designed transformer cores, the reluctance is very small if the core is not saturated. Therefore:

$$F_{net} = N_p i_p - N_s i_s \approx 0$$

$$N_p i_p \approx N_s i_s \Rightarrow \frac{i_p}{i_s} \approx \frac{N_s}{N_p} = \frac{1}{a}$$

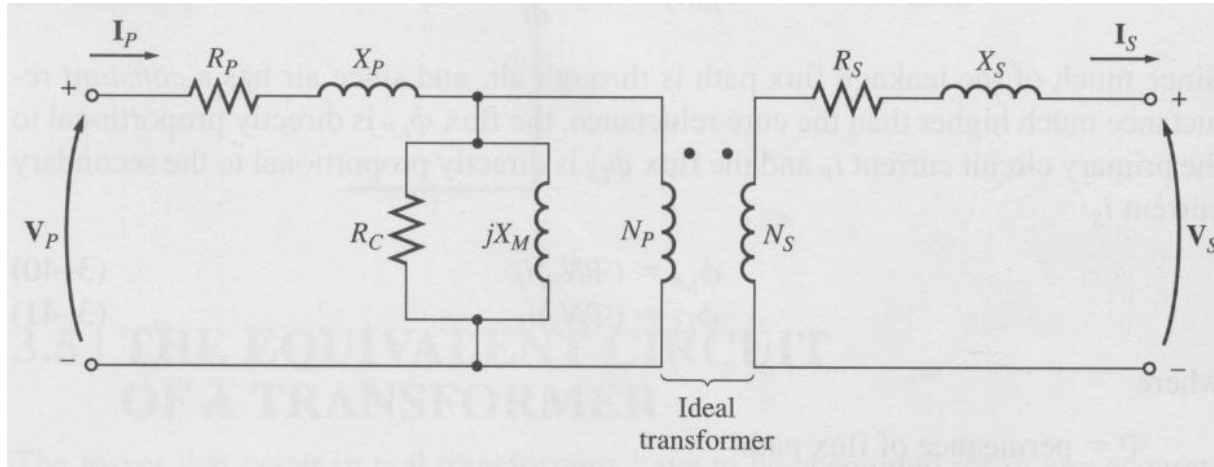


The transformer's equivalent circuit

To model a real transformer accurately, we need to account for the following losses:

1. Copper losses – resistive heating in the windings: I^2R .
2. Eddy current losses – resistive heating in the core: proportional to the square of voltage applied to the transformer.
3. Hysteresis losses – energy needed to rearrange magnetic domains in the core: nonlinear function of the voltage applied to the transformer.
4. Leakage flux – flux that escapes from the core and flux that passes through one winding only.

The exact equivalent circuit of a real transformer

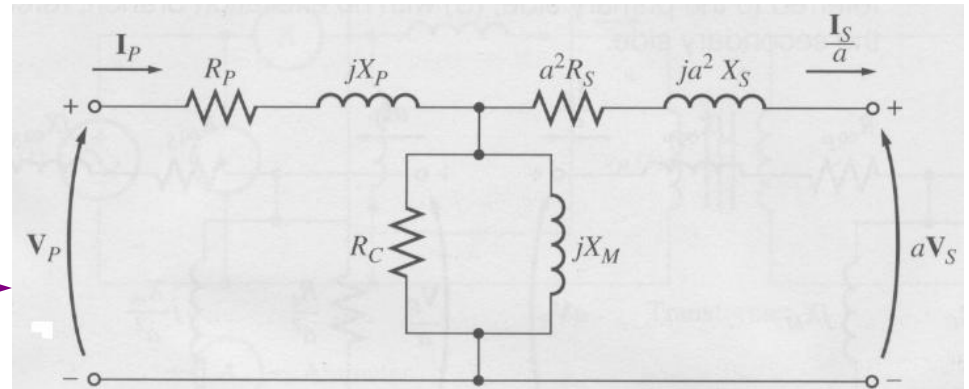


- Copper losses are modeled by the resistors R_p and R_s .
- The leakage flux can be modeled by primary and secondary inductors.
- The magnetization current can be modeled by a reactance X_M connected across the primary voltage source.
- The core-loss current can be modeled by a resistance R_C connected across the primary voltage source.
- Both magnetizing and core loss currents are nonlinear; therefore, X_M and R_C are just approximations.

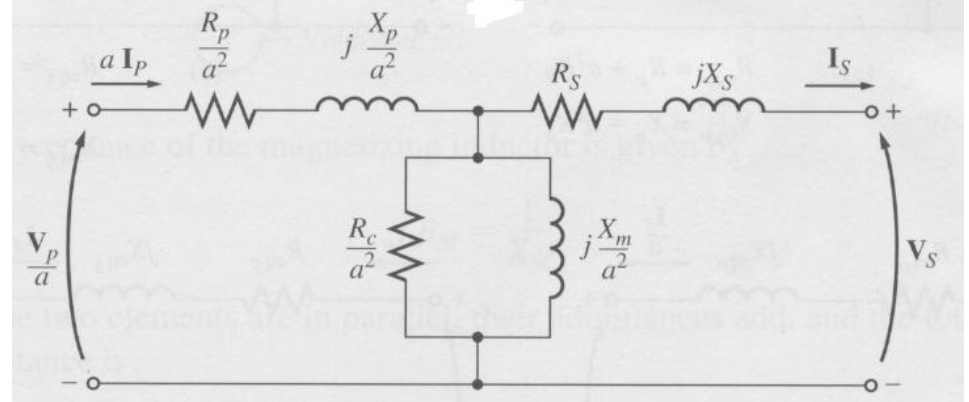
The exact equivalent circuit of a real transformer

The equivalent circuit is usually referred to the primary side or the secondary side of the transformer.

Equivalent circuit of the transformer referred to its primary side.



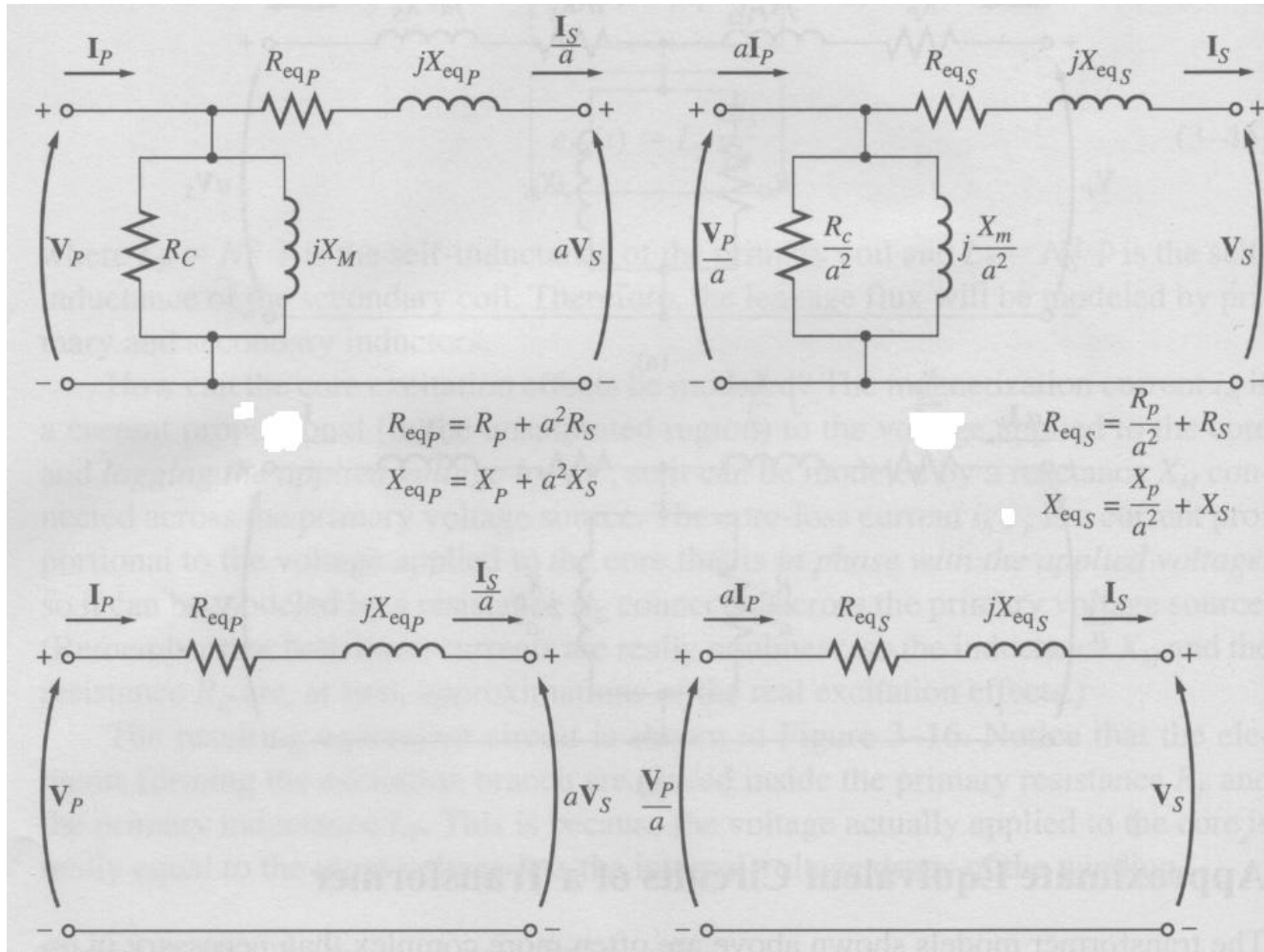
Equivalent circuit of the transformer referred to its secondary side.



Approximate equivalent circuit of a transformer

Referred to the primary side.

Referred to the secondary side



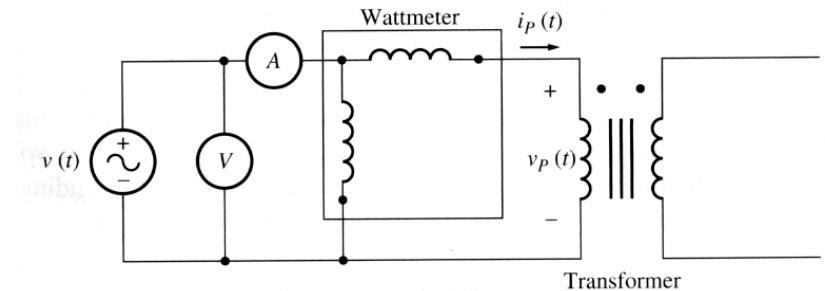
Without an excitation branch referred to the primary side.

Without an excitation branch referred to the secondary side.

Determining the values of components

The open-circuit test.

Full line voltage is applied to the primary side of the transformer. The input voltage, current, and power are measured.



From this information, the power factor of the input current and the magnitude and the angle of the excitation impedance can be determined.

To evaluate R_C and X_M , we define the conductance of the core-loss resistance and The susceptance of the magnetizing inductor :

$$G_C = \frac{1}{R_C}$$

$$B_M = \frac{1}{X_M}$$

Determining the values of components

Since both elements are in parallel, their admittances add. Therefore, the total excitation admittance is:

$$Y_E = G_C - jB_M = \frac{1}{R_C} - j\frac{1}{X_M}$$

The magnitude of the excitation admittance in the open-circuit test is:

$$|Y_E| = \frac{I_{oc}}{V_{oc}}$$

The angle of the admittance in the open-circuit test can be found from the circuit power factor (PF):

$$\cos \theta = PF = \frac{P_{oc}}{V_{oc} I_{oc}}$$

Determining the values of components

In real transformers, the power factor is always lagging, so the angle of the current always lags the angle of the voltage by θ degrees. The admittance is:

$$Y_E = \frac{I_{oc}}{V_{oc}} \angle -\theta = \frac{I_{oc}}{V_{oc}} \angle -\cos^{-1} PF$$

Therefore, it is possible to determine values of R_C and X_M in the open-circuit test.

Determining the values of components

The short-circuit test:

Fairly low input voltage is applied to the primary side of the transformer. This voltage is adjusted until the current in the secondary winding equals to its rated value.

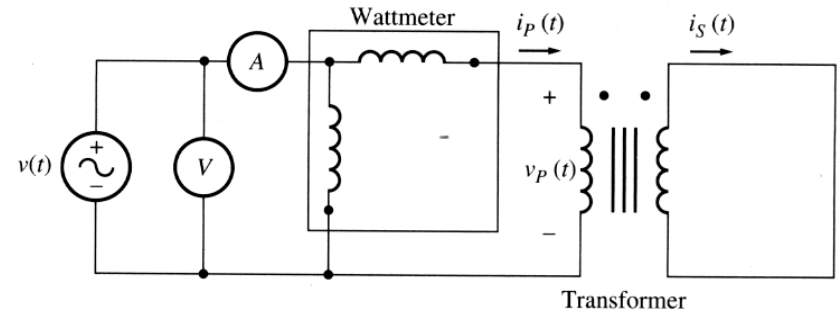
The input voltage, current, and power are measured.

Since the input voltage is low, the current flowing through the excitation branch is negligible; therefore, all the voltage drop in the transformer is due to the series elements in the circuit. The magnitude of the series impedance referred to the primary side of the transformer is:

$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}}$$

The power factor of the current is given by:

$$PF = \cos \theta = \frac{P_{SC}}{V_{SC} I_{SC}}$$



Determining the values of components

Therefore:

$$Z_{SE} = \frac{V_{SC} \angle 0^\circ}{I_{SC} \angle -\theta^\circ} = \frac{V_{SC}}{I_{SC}} \angle \theta^\circ$$

Since the serial impedance Z_{SE} is equal to

$$Z_{SE} = R_{eq} + jX_{eq}$$
$$Z_{SE} = (R_p + a^2 R_s) + j(X_p + a^2 X_s)$$

The same tests can be performed on the secondary side of the transformer. The results will yield the equivalent circuit impedances referred to the secondary side of the transformer.

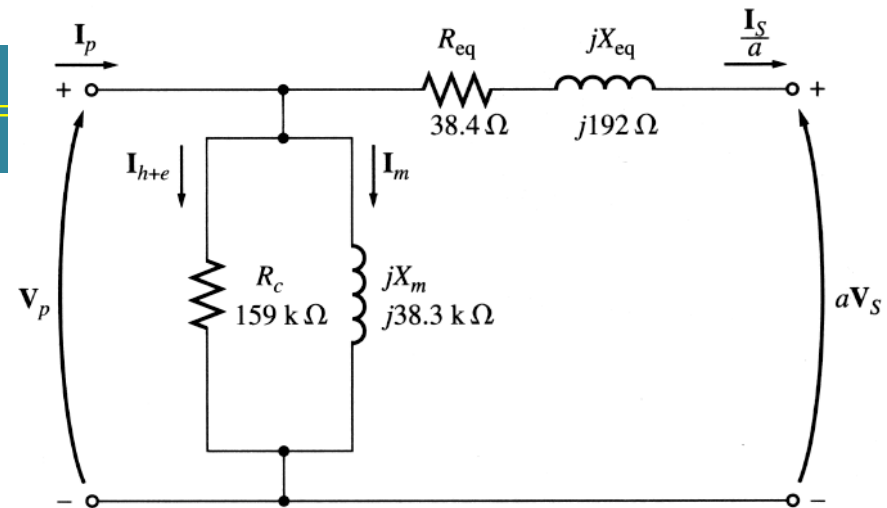
Example

Example 4.2: We need to determine the equivalent circuit impedances of a 20 kVA, 8000/240 V, 60 Hz transformer. The open-circuit and short-circuit tests led to the following data:

$V_{OC} = 8000 \text{ V}$	$V_{SC} = 489 \text{ V}$
$I_{OC} = 0.214 \text{ A}$	$I_{SC} = 2.5 \text{ A}$
$P_{OC} = 400 \text{ W}$	$P_{SC} = 240 \text{ W}$

$$R_C = \frac{1}{0.0000063} = 159 \text{ k}\Omega; \quad X_M = \frac{1}{0.0000261} =$$

$$R_{eq} = 38.3 \Omega; \quad X_{eq} = 192 \Omega$$



The per-unit system

1. Note that the apparent power rating (base) of a transformer is the same for both the primary and secondary sides.
2. On the other hand, the voltage rating (base) changes according to its turn ratio.
3. The impedance of the transformer is often given in pu.

Example: a 110/440 V, 2.5 kVA transformer has a leakage reactance of 1.24% (or 0.0124 pu). Determine the Ohmic value when referred to the low-voltage side:

Ans. 60 mΩ.

The per-unit system: Example

Example 4.4: Sketch the appropriate per-unit equivalent circuit for a 8000/240 V, 60 Hz, 20 kVA transformer with $R_C = 159 \text{ k}\Omega$, $X_M = 38.4 \text{ k}\Omega$, $R_{eq} = 38.3 \text{ }\Omega$, $X_{eq} = 192 \text{ }\Omega$. All the impedance values are referred to the primary side

To convert the transformer to per-unit system, the primary circuit base impedance needs to be found.

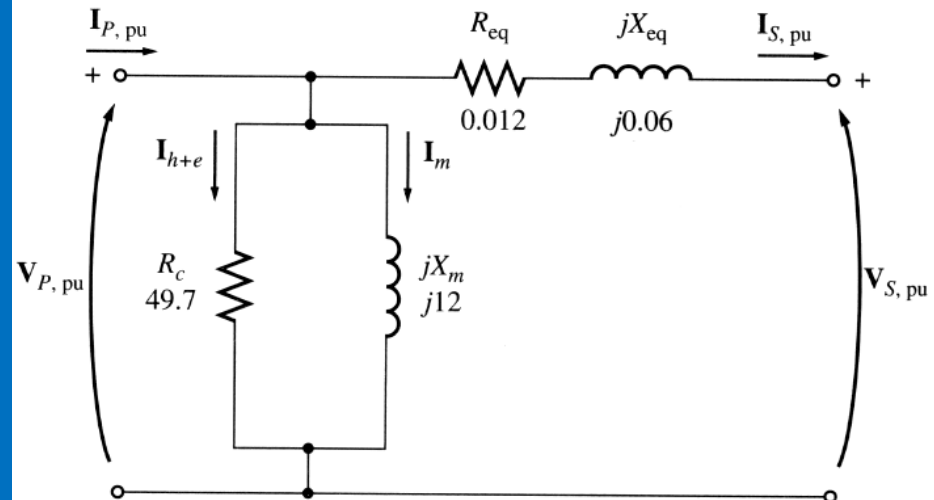
$$V_{base1} = 8000 \text{ V}; \quad S_{base1} = 20000 \text{ VA}$$

$$Z_{base1} = \frac{V_{base1}^2}{S_{base1}} = \frac{8000^2}{20000} = 3200 \text{ }\Omega$$

$$Z_{SE,pu} = \frac{38.4 + j192}{3200} = 0.012 + j0.06 \text{ pu}$$

$$R_{C,pu} = \frac{159000}{3200} = 49.7 \text{ pu}$$

$$X_{M,pu} = \frac{38400}{3200} = 12 \text{ pu}$$



Voltage Regulation (VR)

Since a real transformer contains series impedances, the transformer's output voltage varies with the load even if the input voltage is constant. To compare transformers in this respect, the quantity called a full-load voltage regulation (VR) is defined as follows:

$$VR = \frac{V_{s,nl} - V_{s,fl}}{V_{s,fl}} \cdot 100\% = \frac{V_p / a - V_{s,fl}}{V_{s,fl}} \cdot 100\%$$

In a per-unit system:

$$VR = \frac{V_{p,pu} - V_{s,fl,pu}}{V_{s,fl,pu}} \cdot 100\%$$

Where $V_{s,nl}$ and $V_{s,fl}$ are the secondary no load and full load voltages.

Note: the VR of an ideal transformer is zero under any load condition.

Transformer phasor diagram

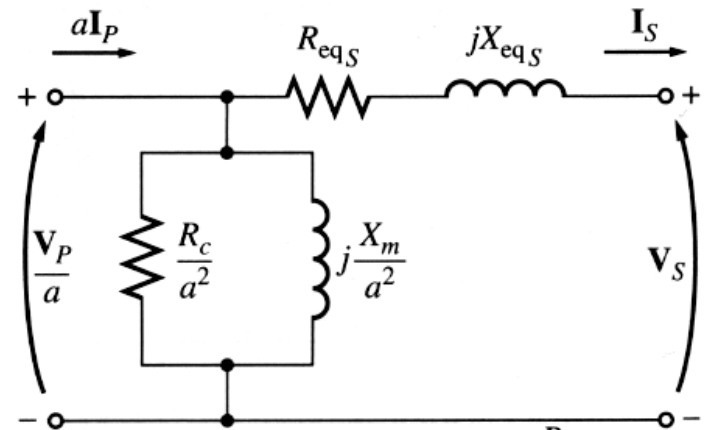
To determine the VR of a transformer, it is necessary to understand the voltage drops within it. Usually, the effects of the excitation branch on transformer VR can be ignored and, therefore, only the series impedances need to be considered. The VR depends on the magnitude of the impedances and on the current phase angle.

A phasor diagram is often used in the VR determinations. The phasor voltage V_s is assumed to be at 0° and all other voltages and currents are compared to it.

Considering the diagram and by applying the Kirchhoff's voltage law, the primary voltage is:

$$\frac{V_p}{a} = V_s + R_{eq} I_s + jX_{eq} I_s$$

A transformer phasor diagram is a graphical representation of this equation.



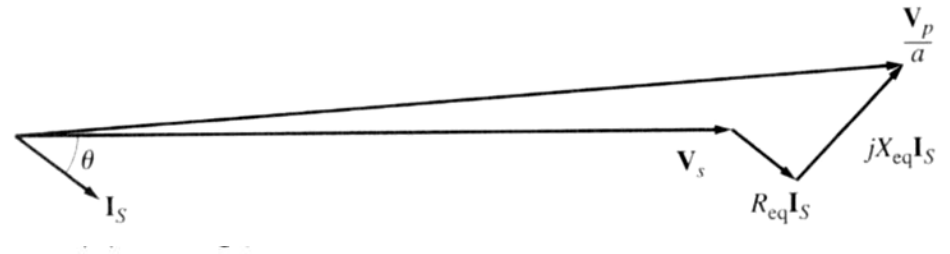
$$R_{eqS} = \frac{R_p}{a^2} + R_s$$

$$X_{eqS} = \frac{X_p}{a^2} + X_s$$

Transformer phasor diagram

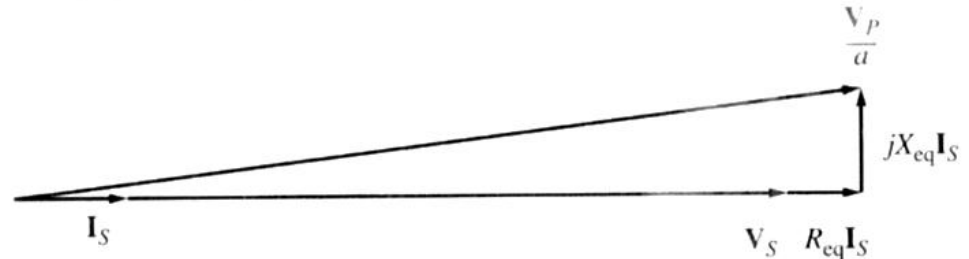
A transformer operating at a lagging power factor:

It is seen that $V_p/a > V_s$, $VR > 0$



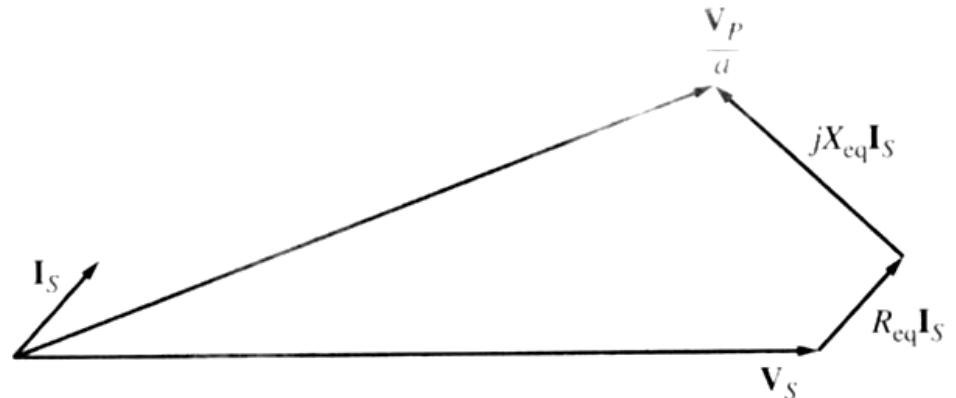
A transformer operating at a unity power factor:

It is seen that $VR > 0$



A transformer operating at a leading power factor:

If the secondary current is leading, the secondary voltage can be higher than the referred primary voltage; $VR < 0$.



Transformer efficiency

The efficiency of a transformer is defined as:

$$\eta = \frac{P_{out}}{P_{in}} \cdot 100\% = \frac{P_{out}}{P_{out} + P_{loss}} \cdot 100\%$$

Note: the same equation describes the efficiency of motors and generators.

Considering the transformer equivalent circuit, we notice three types of losses:

1. Copper (I^2R) losses – are accounted for by the series resistance
2. Hysteresis and eddy current losses – are accounted for by the resistor R_c .

Since the output power is

$$P_{out} = V_s I_s \cos \theta_s$$

The transformer efficiency is

$$\eta = \frac{V_s I_s \cos \theta}{P_{Cu} + P_{core} + V_s I_s \cos \theta} \cdot 100\%$$

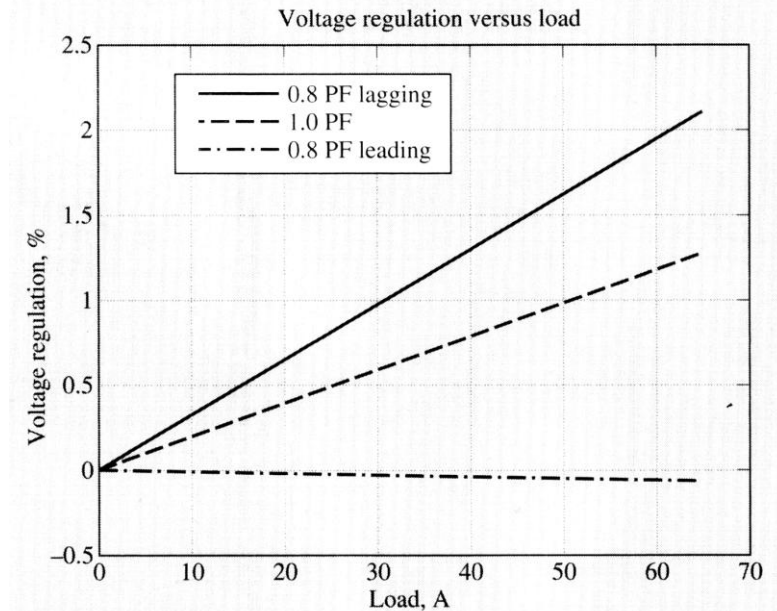
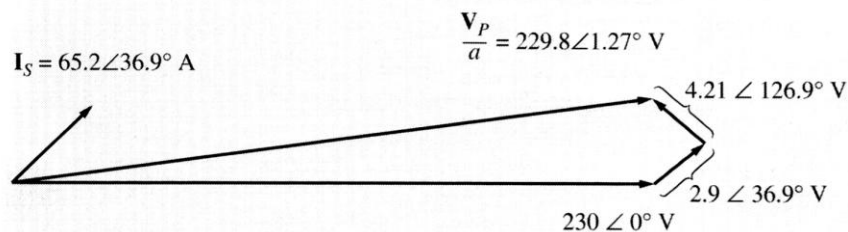
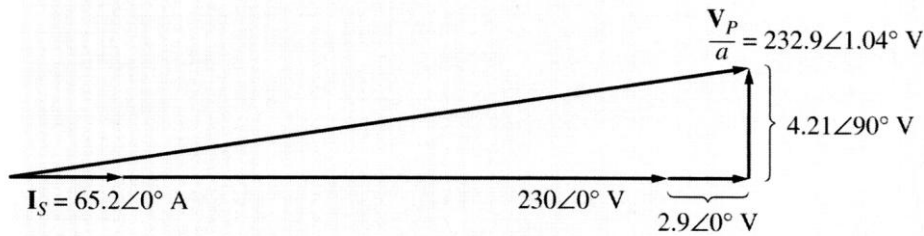
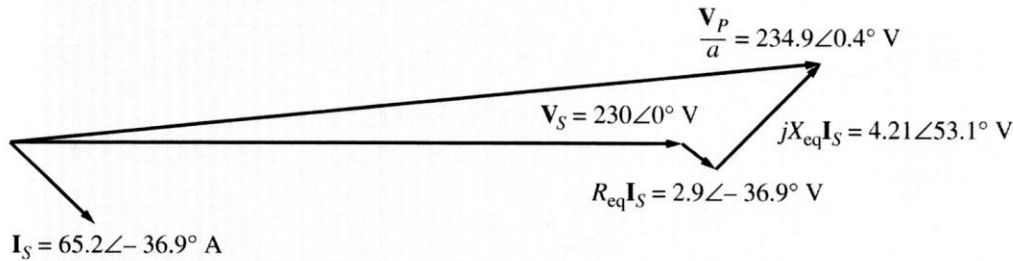
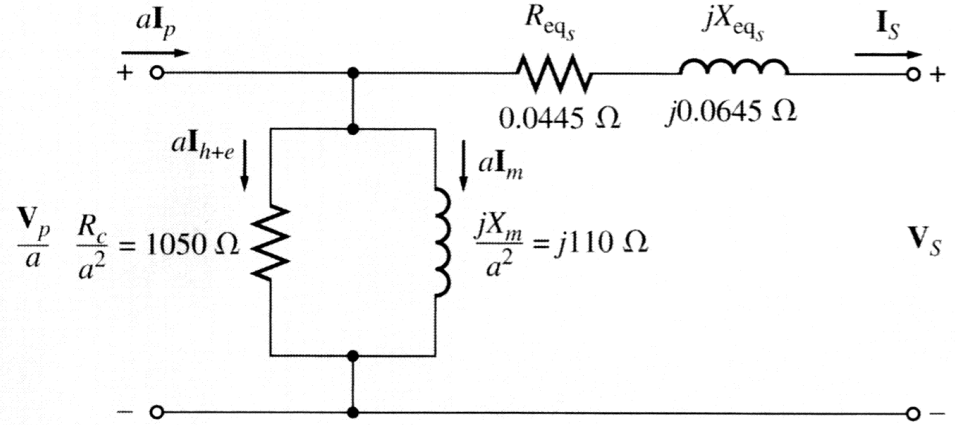
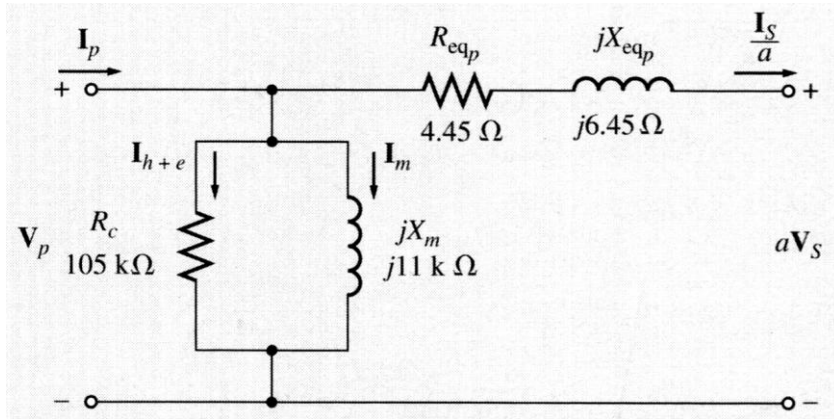
The transformer efficiency: Example

Example 4.5: A 15 kVA, 2300/230 V transformer was tested to by open-circuit and closed-circuit tests. The following data was obtained:

$V_{OC} = 2300 \text{ V}$	$V_{SC} = 47 \text{ V}$
$I_{OC} = 0.21 \text{ A}$	$I_{SC} = 6.0 \text{ A}$
$P_{OC} = 50 \text{ W}$	$P_{SC} = 160 \text{ W}$

- Find the equivalent circuit of this transformer referred to the high-voltage side.
- Find the equivalent circuit of this transformer referred to the low-voltage side.
- Calculate the full-load voltage regulation at 0.8 lagging power factor, at 1.0 power factor, and at 0.8 leading power factor.
- Plot the voltage regulation as load is increased from no load to full load at power factors of 0.8 lagging, 1.0, and 0.8 leading.
- What is the efficiency of the transformer at full load with a power factor of 0.8 lagging?

The transformer efficiency: Example



$$\eta = \frac{P_{out}}{P_{Cu} + P_{core} + P_{out}} \cdot 100\% = 98.03\%$$

Transformer taps and voltage regulation

We assumed before that the transformer turns ratio is a fixed (constant) for the given transformer. Frequently, distribution transformers have a series of taps in the windings to permit small changes in their turns ratio. Typically, transformers may have 4 taps in addition to the nominal setting with spacing of 2.5 % of full-load voltage. Therefore, adjustments up to 5 % above or below the nominal voltage rating of the transformer are possible.

Example 4.6: A 500 kVA, 13 200/480 V transformer has four 2.5 % taps on its primary winding. What are the transformer's voltage ratios at each tap setting?

<i>Ans:</i>	+ 5.0% tap	13 860/480 V
	+ 2.5% tap	13 530/480 V
	Nominal rating	13 200/480 V
	- 2.5% tap	12 870/480 V
	- 5.0% tap	12 540/480 V

Transformer taps and voltage regulation

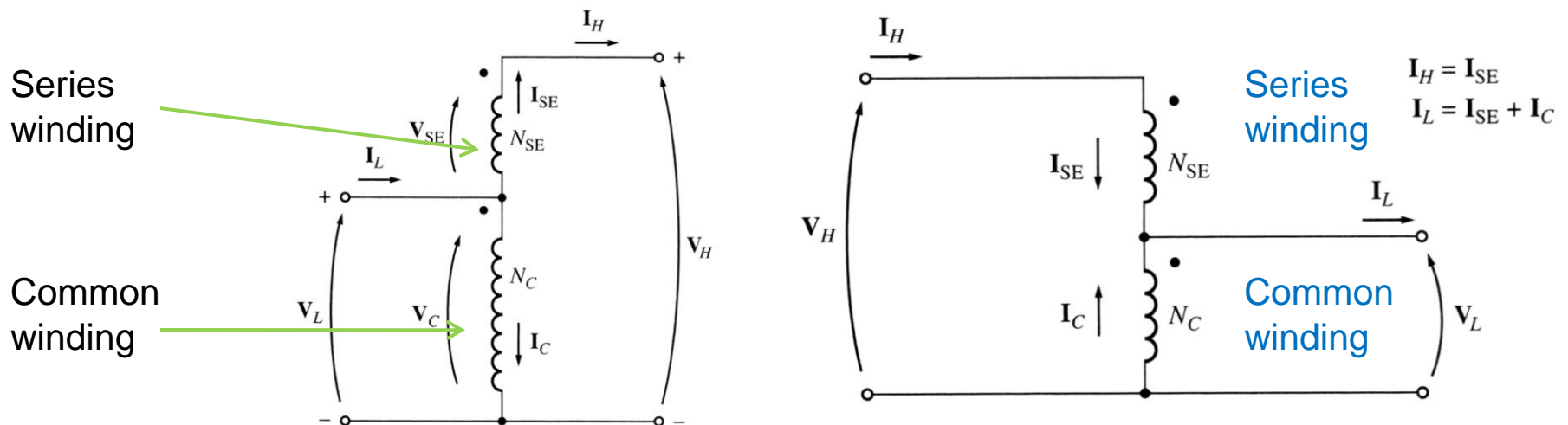
- Taps allow adjustment of the transformer in the field to accommodate for local voltage variations.
- Sometimes, transformers are used on a power line, whose voltage varies widely with the load (due to high line impedance, for instance). Normal loads need fairly constant input voltage though
- One possible solution to this problem is to use a special transformer called a tap changing under load (TCUL) transformer or voltage regulator. TCUL is a transformer with the ability to change taps while power is connected to it. A voltage regulator is a TCUL with build-in voltage sensing circuitry that automatically changes taps to keep the system voltage constant.
- These “self-adjusting” transformers are very common in modern power systems.

The autotransformer

Sometimes, it is desirable to change the voltage by a small amount (for instance, when the consumer is far away from the generator and it is needed to raise the voltage to compensate for voltage drops).

In such situations, it would be expensive to wind a transformer with two windings of approximately equal number of turns. An autotransformer (a transformer with only one winding) is used instead.

Diagrams of step-up and step-down autotransformers:



Output (up) or input (down) voltage is a sum of voltages across common and series windings.

The autotransformer

Since the autotransformer's coils are physically connected, a different terminology is used for autotransformers:

The voltage across the common winding is called a common voltage V_C , and the current through this coil is called a common current I_C . The voltage across the series winding is called a series voltage V_{SE} , and the current through that coil is called a series current I_{SE} .

The voltage and current on the low-voltage side are called V_L and I_L ; the voltage and current on the high-voltage side are called V_H and I_H .

For the autotransformers:

$$\frac{V_C}{V_{SE}} = \frac{N_C}{N_{SE}}$$

$$N_C I_C = N_{SE} I_{SE}$$

$$\frac{V_L}{V_H} = \frac{N_C}{N_C + N_{SE}}$$

$$\frac{I_L}{I_H} = \frac{N_C + N_{SE}}{N_C}$$

The apparent power advantage

The ratio of the apparent power in the primary and secondary of the autotransformer to the apparent power **actually** traveling through its windings is

$$\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_C}{N_{SE}}$$

The last equation described the apparent power rating advantage of an autotransformer over a conventional transformer.

S_W is the apparent power actually passing through the windings. The rest passes from primary to secondary parts without being coupled through the windings.

Note that the smaller the series winding, the greater the advantage!

The apparent power advantage

For example, a 5 MVA autotransformer that connects a 110 kV system to a 138 kV system would have a turns ratio (common to series) 110:28. Such an autotransformer would actually have windings rated at:

$$S_W = S_{IO} \frac{N_{SE}}{N_{SE} + N_C} = 5 \cdot \frac{28}{28 + 110} = 1.015 \text{ MVA}$$

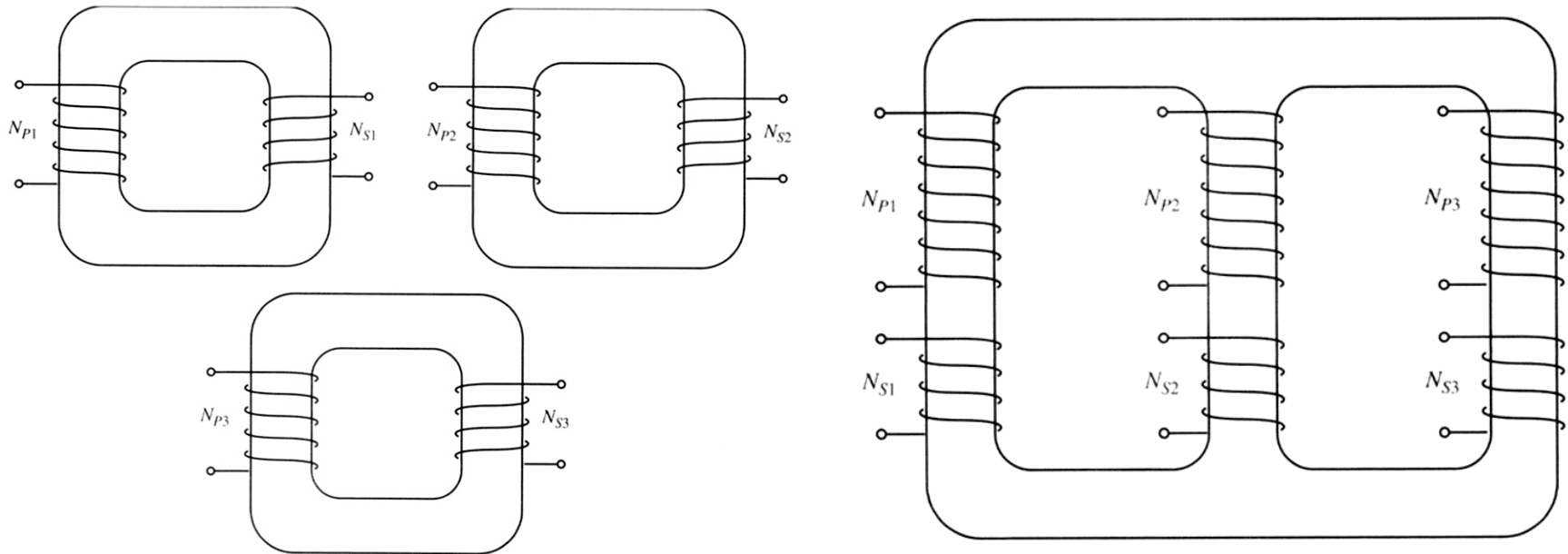
Therefore, the autotransformer would have windings rated at slightly over 1 MVA instead of 5 MVA, which makes it 5 times smaller and, therefore, considerably less expensive.

However, the construction of autotransformers is usually slightly different. In particular, the insulation on the smaller coil (the series winding) of the autotransformer is made as strong as the insulation on the larger coil to withstand the full output voltage.

The primary disadvantage of an autotransformer is that there is a direct physical connection between its primary and secondary circuits. Therefore, the electrical isolation of two sides is lost.

3-phase transformers

The majority of the power generation/distribution systems in the world are 3-phase systems. The transformers for such circuits can be constructed either as a 3-phase bank of independent identical transformers (can be replaced independently) or as a single transformer wound on a single 3-legged core (lighter, cheaper, more efficient).



3-phase transformer connections

We assume that any single transformer in a 3-phase transformer (bank) behaves exactly as a single-phase transformer. The impedance, voltage regulation, efficiency, and other calculations for 3-phase transformers are done on a per-phase basis, using the techniques studied previously for single-phase transformers.

Four possible connections for a 3-phase transformer bank are:

1. Y-Y
2. Y- Δ
3. Δ - Δ
4. Δ -Y

3-phase transformer connections

The primary voltage on each phase of the transformer is

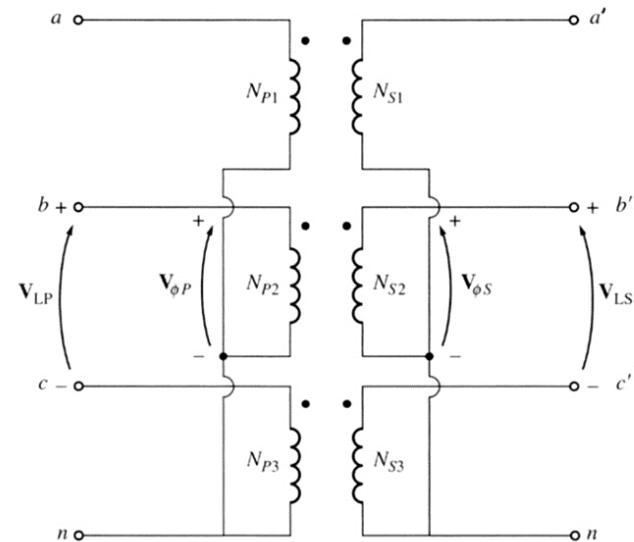
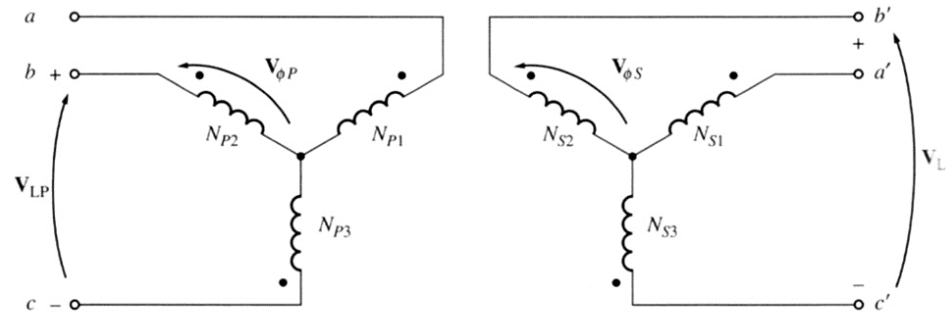
$$V_{\phi P} = \frac{V_{LP}}{\sqrt{3}}$$

The secondary phase voltage is

$$V_{LS} = \sqrt{3}V_{\phi S}$$

The overall voltage ratio is

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{\sqrt{3}V_{\phi S}} = a$$



3-phase transformer connections

The primary voltage on each phase of the transformer is

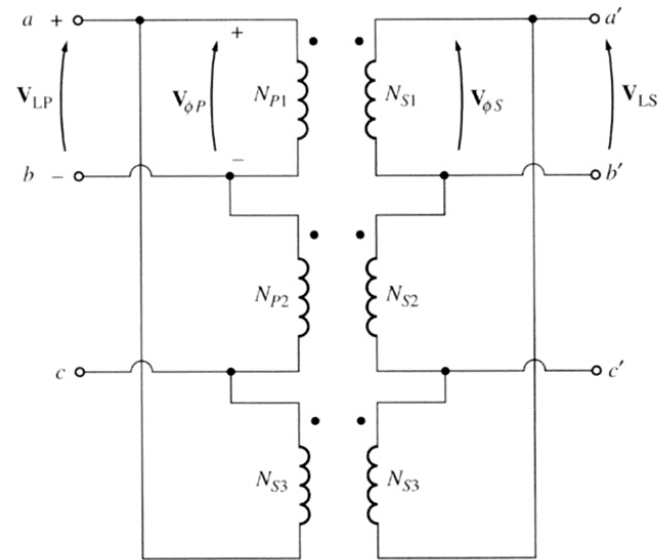
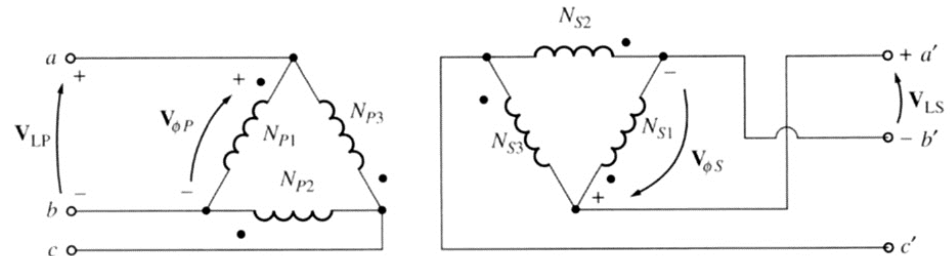
$$V_{\phi P} = V_{LP}$$

The secondary phase voltage is

$$V_{LS} = V_{\phi S}$$

The overall voltage ratio is

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3-phase transformer connections

The primary voltage on each phase of the transformer is

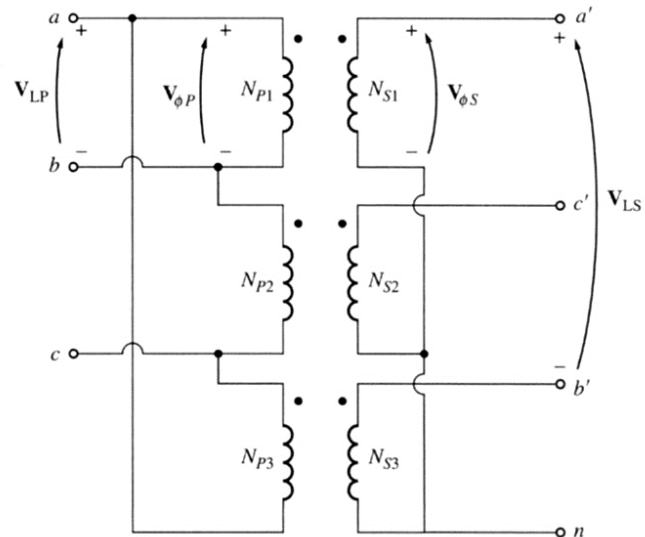
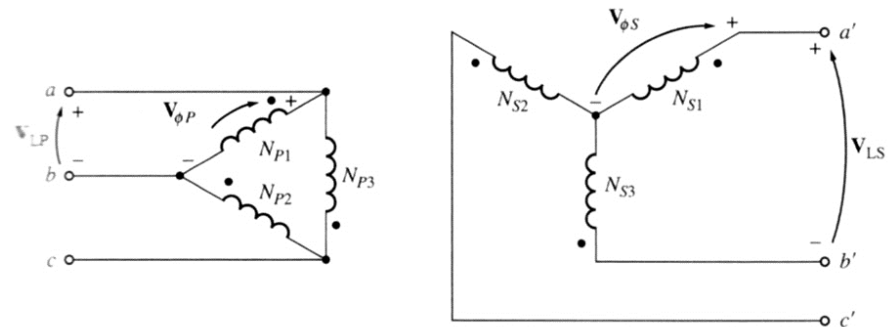
$$V_{\phi P} = V_{LP}$$

The secondary phase voltage is

$$V_{LS} = \sqrt{3}V_{\phi S}$$

The overall voltage ratio is

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{\sqrt{3}V_{\phi S}} = \frac{a}{\sqrt{3}}$$



Transferring Ohmic Values of Per-Phase Impedances

Y-Y

$N_1 : N_2$

$$\left| \frac{V_{LN}}{V_{in}} \right| = \frac{N_1}{N_2} ; \left| \frac{V_{LL}}{V_{ll}} \right| = \frac{N_1}{N_2}$$

$$Z_H = \left(\frac{N_1}{N_2} \right)^2 Z_L = \left| \frac{V_{LL}}{V_{ll}} \right|^2 Z_L$$

Y-Δ

$N_1 : N_2/\sqrt{3}$

$$\left| \frac{V_{LN}}{V_{ll}} \right| = \frac{N_1}{N_2} ; \left| \frac{V_{LL}}{V_{ll}} \right| = \sqrt{3} \frac{N_1}{N_2}$$

$$Z_H = \left(\frac{N_1}{N_2/\sqrt{3}} \right)^2 Z_L = \left| \frac{V_{LL}}{V_{ll}} \right|^2 Z_L$$

Δ-Y

$N_1/\sqrt{3} : N_2$

$$\left| \frac{V_{LL}}{V_{in}} \right| = \frac{N_1}{N_2} ; \left| \frac{V_{LL}}{V_{ll}} \right| = \frac{1}{\sqrt{3}} \frac{N_1}{N_2}$$

$$Z_H = \left(\frac{N_1/\sqrt{3}}{N_2} \right)^2 Z_L = \left| \frac{V_{LL}}{V_{ll}} \right|^2 Z_L$$

Δ-Δ

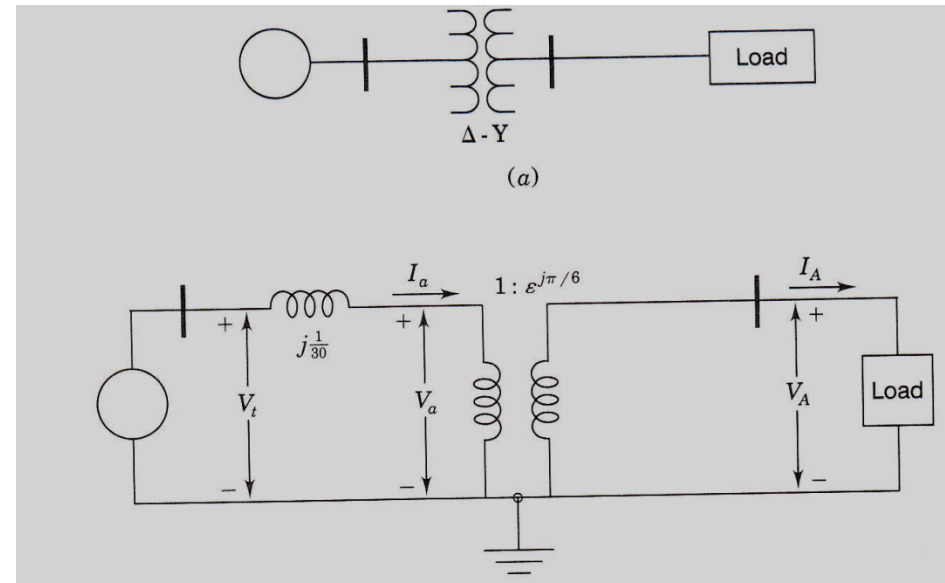
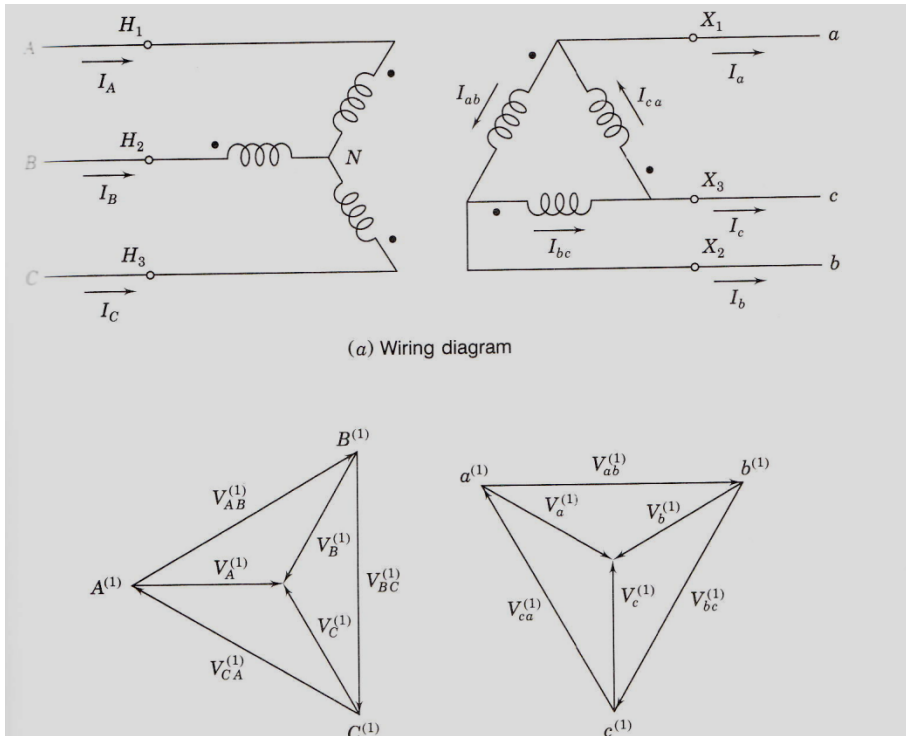
$N_1/\sqrt{3} : N_2/\sqrt{3}$

$$\left| \frac{V_{LN}}{V_{in}} \right| = \frac{N_1/\sqrt{3}}{N_2/\sqrt{3}} ; \left| \frac{V_{LL}}{V_{ll}} \right| = \frac{N_1}{N_2}$$

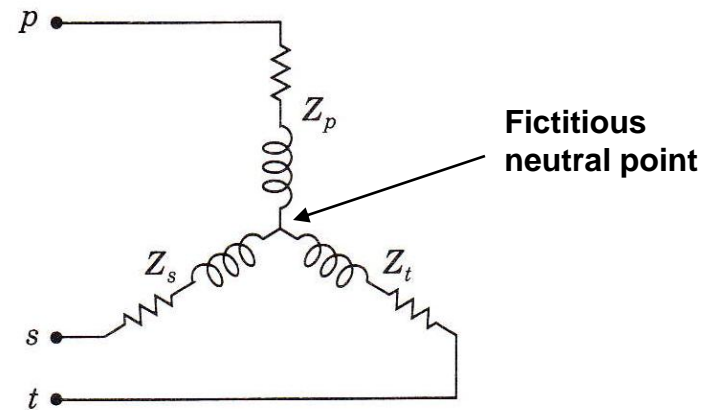
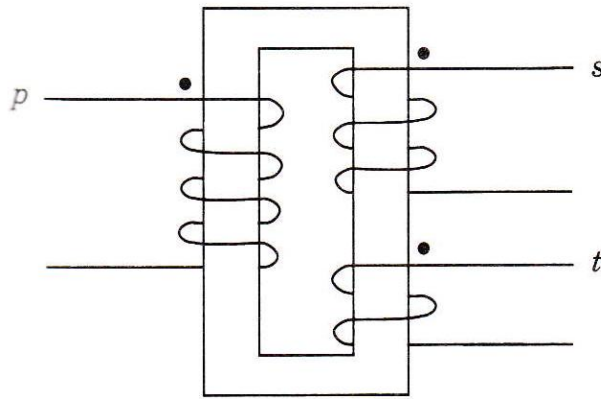
$$Z_H = \left(\frac{N_1/\sqrt{3}}{N_2/\sqrt{3}} \right)^2 Z_L = \left| \frac{V_{LL}}{V_{ll}} \right|^2 Z_L$$

Phase Shifts and Equivalent Circuits

- When stepping up the voltage in a Y- Δ or Δ -Y transformer, the positive sequence voltages and currents are advanced by 30° .



Per-Unit Impedances in 3-Winding Transformer



Z_p , Z_s , and Z_t , are the impedances of the primary, secondary, and tertiary windings, respectively, referred to the primary circuit. if Z_{ps} , Z_{pt} , and Z_{st} , are the measured impedances referred to the primary circuit, then

$$Z_{ps} = Z_p + Z_s$$

$$Z_p = \frac{1}{2}(Z_{ps} + Z_{pt} - Z_{st})$$

$$Z_{pt} = Z_p + Z_t$$

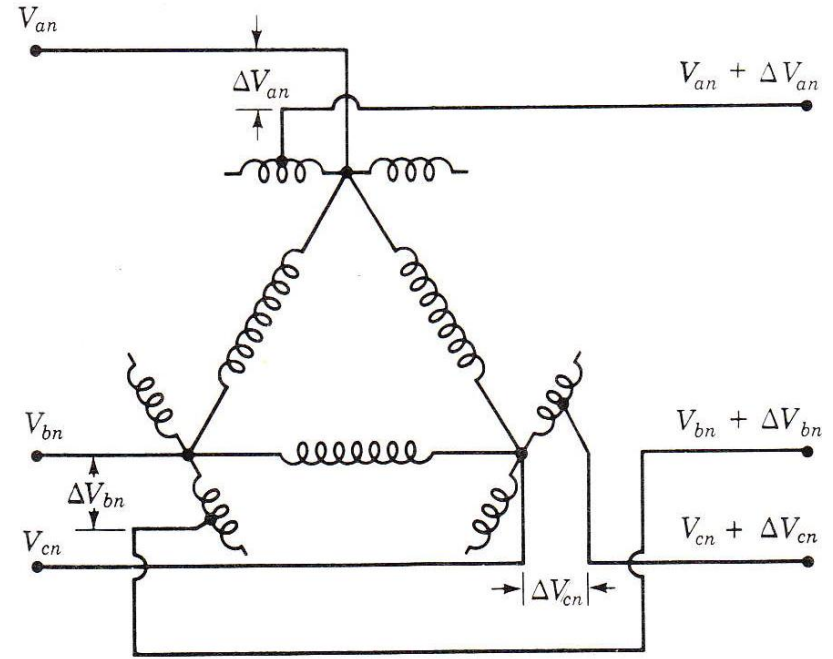
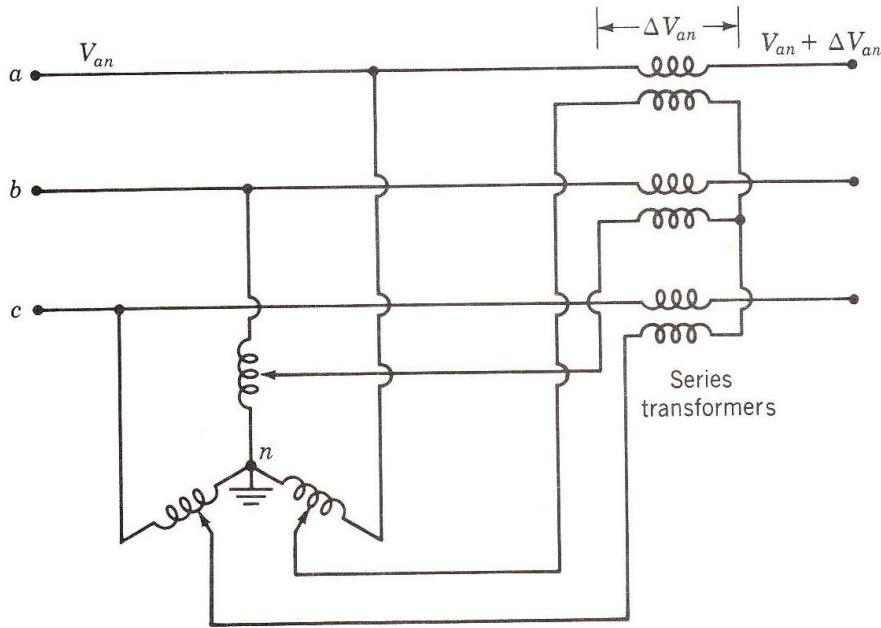


$$Z_s = \frac{1}{2}(Z_{ps} + Z_{st} - Z_{pt})$$

$$Z_{st} = Z_s + Z_t$$

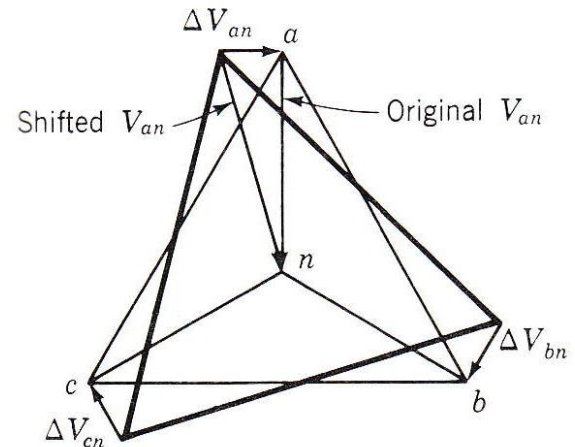
$$Z_t = \frac{1}{2}(Z_{pt} + Z_{st} - Z_{ps})$$

Regulating Transformers for Control of Voltage Magnitude and Phase

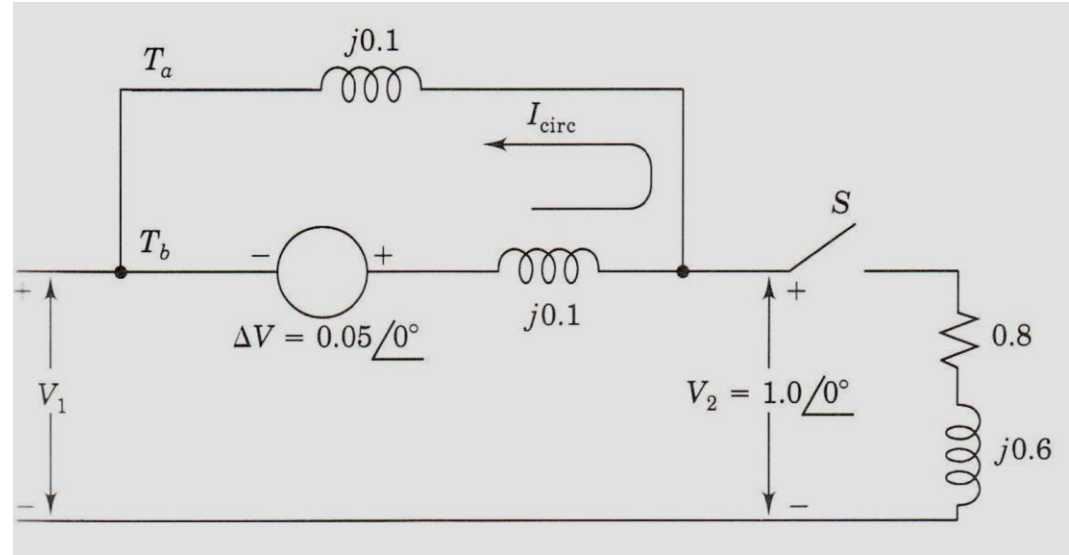
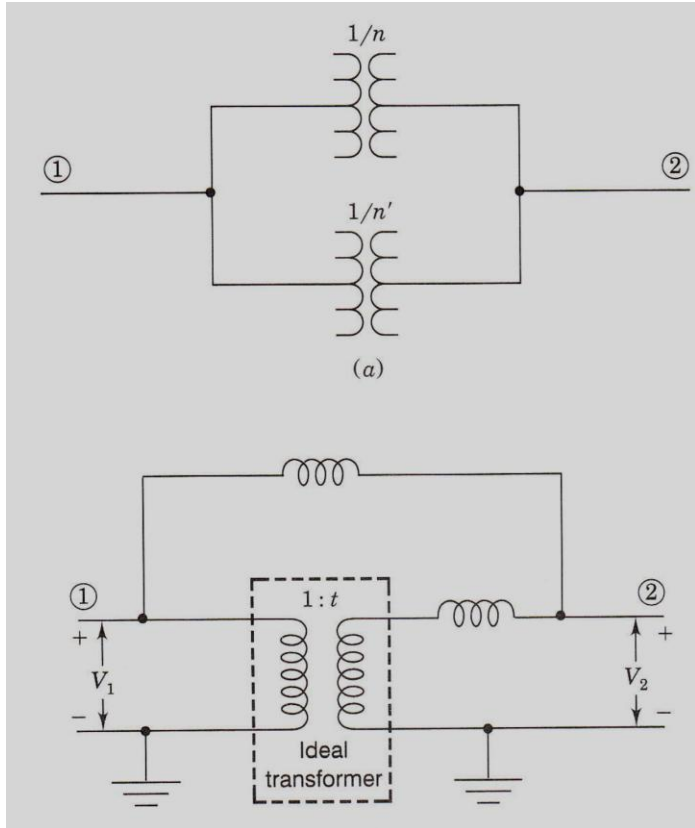


Control of Voltage Magnitude

Control of Voltage Phase Angle



Example (refer to book)



Case 0: Two parallel transformers with the same turn ratio and same rating. → the transformers share the load equally

Case 1: T_b is providing a voltage ratio 5% higher than T_a . → The transformer with the higher tap setting is supplying most of the reactive power to the load.

Case 2: T_b is a regulating transformer with a phase shift of 3° . → The phase shifting transformer is supplying most of the real power to the load.

Transformers Operating in Parallel

- Example: a 100 kVA transformer (with $X = 4\%$) is connected in parallel with an existing 250 kVA transformer (with $X = 6\%$). The transformers are rated at 7200V/240V, and they supply a load of 330 kVA at nominal voltage.

Calculate the following:

1. The nominal secondary current of each transformer: (Ans. 416.7 A and 1,041A)
2. The current and impedance of the load (Ans. 1,375 A and 174.5 m Ω)
3. The impedance of each transformer when referred to the secondary side: (Ans. 23 m Ω and 13.8 m Ω)
4. The current flow through each transformer secondary winding (Ans. 515.6A and 859.4)
5. Which transformer, if any, is overloaded? (Ans. The smaller transformer is carrying 124 kVA – nearly a 25% overload!)

Transformer ratings: Voltage and Frequency

The **voltage** rating is a) used to protect the winding insulation from breakdown; b) related to the magnetization current of the transformer (more important)

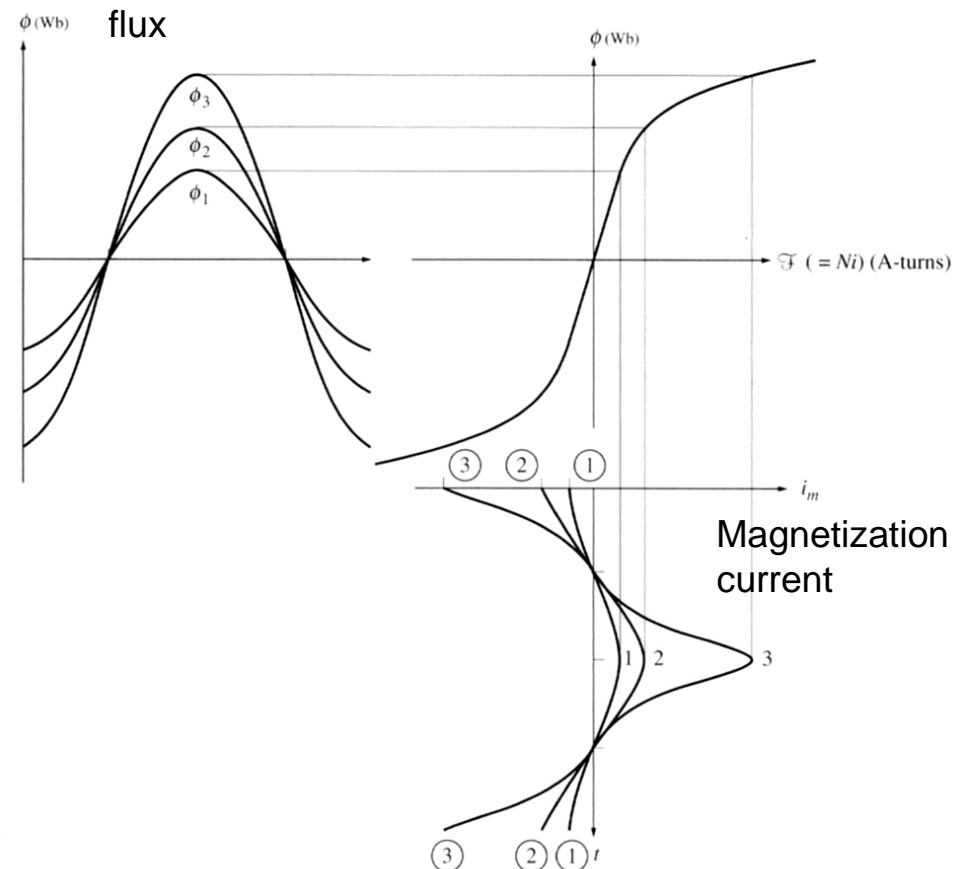
If a steady-state voltage

$$v(t) = V_M \sin \omega t$$

is applied to the transformer's primary winding, the transformer's flux will be

$$\phi(t) = \frac{1}{N_p} \int v(t) dt = -\frac{V_M}{\omega N_p} \cos \omega t$$

An increase in voltage will lead to a proportional increase in flux. However, after some point (in a saturation region), such increase in flux would require an unacceptable increase in magnetization current!



Transformer ratings: Voltage and Frequency

Therefore, the maximum applied voltage (and thus the rated voltage) is set by the maximum acceptable magnetization current in the core.

We notice that the maximum flux is also related to the frequency:

$$\phi_{\max} = \frac{V_{\max}}{\omega N_p}$$

Therefore, to maintain the same maximum flux, a change in frequency (say, 50 Hz instead of 60 Hz) must be accompanied by the corresponding correction in the maximum allowed voltage. This reduction in applied voltage with frequency is called derating. As a result, a 50 Hz transformer may be operated at a 20% higher voltage on 60 Hz if this would not cause insulation damage.

Transformer ratings: Apparent Power

The apparent power rating sets (together with the voltage rating) the current through the windings. The current determines the I^2R losses and, therefore, the heating of the coils.

Overheating shortens the life of transformer's insulation. Under any circumstances, the temperature of the windings must be limited.

A typical power transformer rating would be OA/FA/FOA where

- The OA (open-air) rating is the base power rating.
- The FA (forced-air) rating is the power carrying capability when forced air cooling (fans) are used.
- The FOA (forced-oil-air) rating is the power carrying capability when forced air cooling (fans) are used in addition to oil circulating pumps.

Each cooling method provides nearly an additional 1/3 capability. For example, a common substation transformer is rated at 21/28/35 MVA.

Transformer ratings: Current inrush

Assuming that the following voltage is applied to the transformer at the moment it is connected to the line:

$$v(t) = V_M \sin(\omega t + \theta)$$

The maximum flux reached on the first half-cycle depends on the phase of the voltage at the instant the voltage is applied. If the initial voltage is

$$v(t) = V_M \sin(\omega t + 90^\circ) = V_M \cos \omega t$$

and the initial flux in the core is zero, the maximum flux during the first half-cycle is equal to the maximum steady-state flux (which is ok):

$$\phi_{\max} = \frac{V_M}{\omega N_p}$$

However, if the voltage's initial phase is zero, i.e.

$$v(t) = V_M \sin(\omega t)$$

Transformer ratings: Current inrush

the maximum flux during the first half-cycle will be

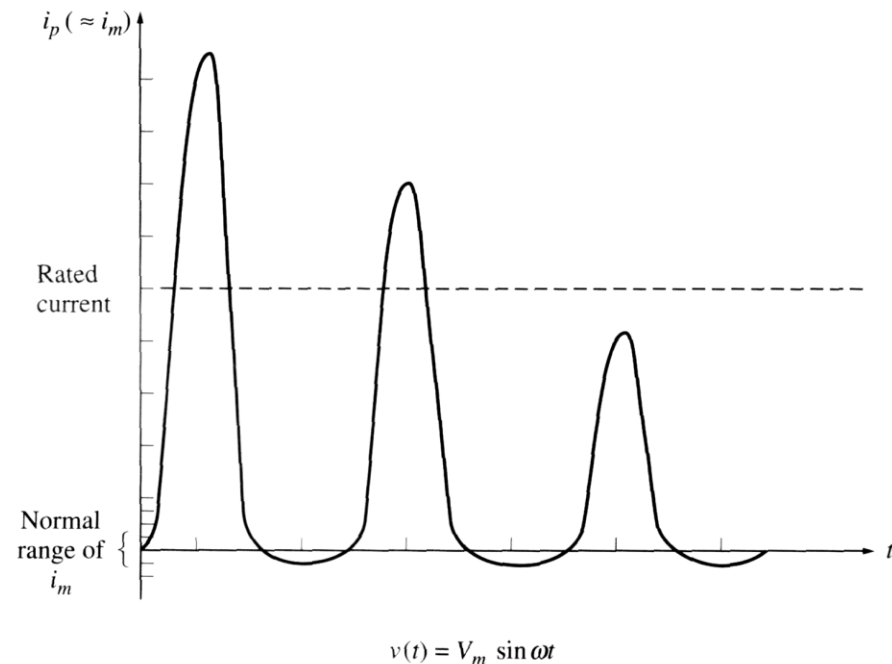
$$\phi_{\max} = \frac{1}{N_p} \int_0^{\pi/\omega} V_M \sin(\omega t) dt = -\frac{V_M}{\omega N_p} \cos(\omega t) \Big|_0^{\pi/\omega} = \frac{2V_M}{\omega N_p}$$

Which is twice higher than a normal steady-state flux!

Doubling the maximum flux in the core can bring the core in a saturation and, therefore, may result in a huge magnetization current!

Normally, the voltage phase angle cannot be controlled. As a result, a large inrush current is possible during the first several cycles after the transformer is turned ON.

The transformer and the power system must be able to handle these currents.



Practice Problems (Chap. 2)

6, 8, 13, 15, 19, 21, 23.